MATH 4613 GALOIS THEORY Problem Set II Name:
PROF. PAUL BAILEY November 15, 2006

Due Date: Monday November 27, 2006.
Write your solutions neatly on separate pieces of paper and attach this sheet to the front.

Problem 1. Let $m, n \in \mathbb{Z}$ be nonzero and suppose that there exist integers $x, y \in \mathbb{Z}$ such that $mx + ny = 1$. Show that $\gcd(m, n) = 1$.

Problem 2. Let $f(x) = x^2 + 46x + 54$. Show that f is irreducible over $\mathbb{Q}$, but is reducible over $\mathbb{Z}_{17}$.

Problem 3. Let $\beta = \sqrt[3]{\sqrt{2} + \sqrt{3}}$.

(a) Find the minimum polynomial of β over $\mathbb{Q}$.

(b) Find the minimum polynomial of β over $\mathbb{Q}[\sqrt{6}]$.

Problem 4. Let $\beta = e^{2\pi/16}$.

(a) Find the minimum polynomial of β over $\mathbb{Q}$.

(b) Find the minimum polynomial of β over $\mathbb{Q}[i]$.

(c) Find the minimum polynomial of β over $\mathbb{Q}[\sqrt{2}]$.

Problem 5. Let $f(x) = x^{12} - 1$, and let $E \subset \mathbb{C}$ be the splitting field of f over $\mathbb{Q}$. Write E as a multiple extension, and find $[E : \mathbb{Q}]$.

Problem 6. Let K/E and E/F be algebraic extensions. Show that K/F is an algebraic extension.